Learning Talagrand DNF Formulas

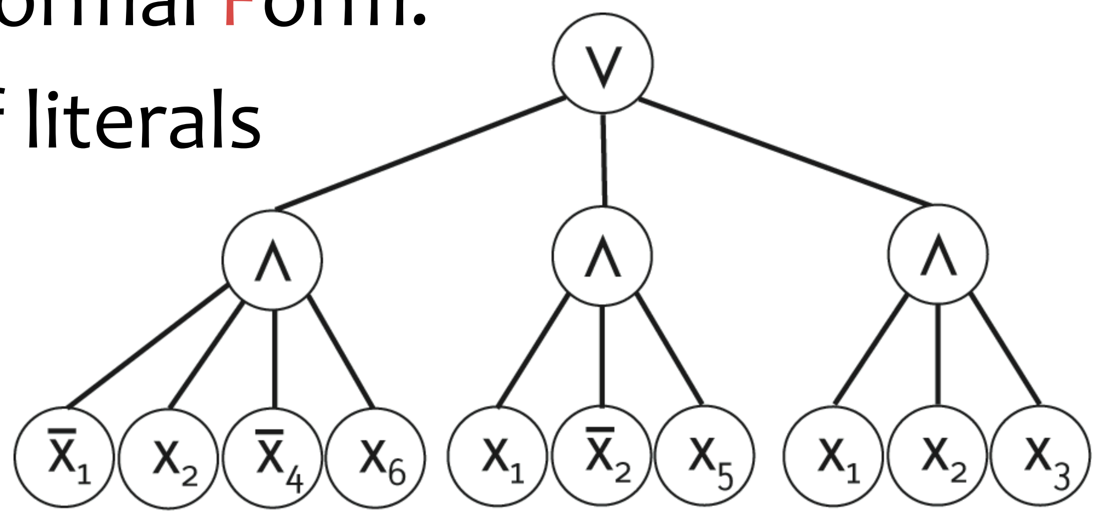
Homin K. Lee

UT-Austin

DNF Formulas

Disjunctive Normal Form:

OR of AND of literals



Can also write as: $\bar{x}_1 x_2 \bar{x}_4 x_6 \vee x_1 \bar{x}_2 x_5 \vee x_1 x_2 x_3$

Size is the number of AND gates (terms).

PAC Learning DNF Formulas

A is a PAC-learner for poly(n)-size DNF if $\forall f$ in the class given uniform random examples $(x, f(x))$ w.h.p. outputs h s.t.

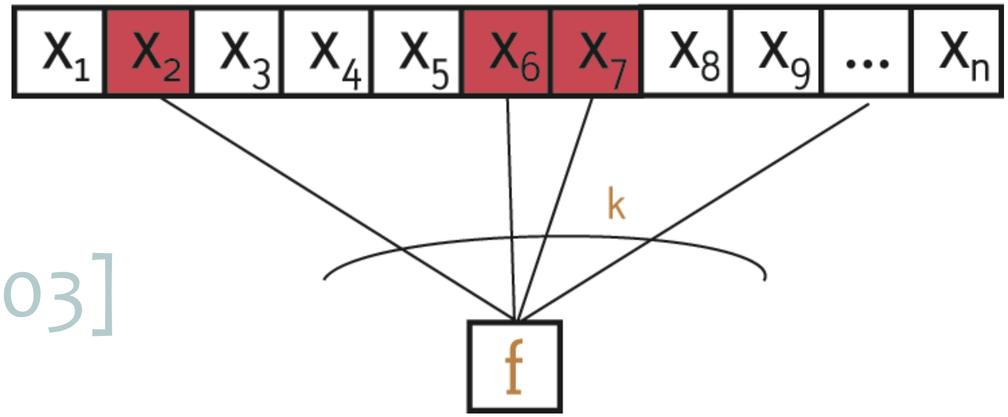
$\Pr[\, h(x) = f(x) \,] \geq 1 - \varepsilon$ [V84]

Best alg takes time $n^{O(\log n/\varepsilon)}$ [V90]

Juntas

Boolean funcs that depend on $\leq$ k vars.

[B03]



Best alg takes time $n^{0.7k}$ [MOS03]

Learning DNF $\Rightarrow$ Learning O(log n) Juntas

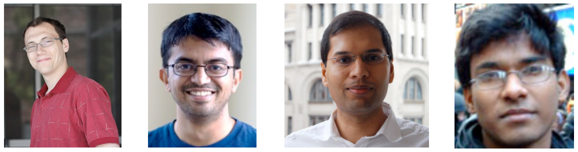
Parity with Noise

$S = \{x_1, x_5, x_8, x_9\}$

$\chi_S(x) = 1$ if odd # of vars in S are set to 1.

$\chi_S(x) \oplus \eta$, $\eta = 1$ w.p. p

Best alg takes time $2^{O(n/\log n)}$ [BKW00]



Learning PWN, $|S| = O(\log n) \Rightarrow$ Learning DNF

[FGKP06]

Statistical Queries

An SQ-oracle given g, outputs a good estimate to $E[g(x,f(x))]$

SQ-learners for DNF take $n^{\omega(1)}$ queries [K93]

Almost all PAC-learning algs are SQ algs!

Monotone DNF

Monotone: no negations on the literals

$x_1x_2x_4x_6 \vee x_1x_2x_5 \vee x_1x_2x_3$

A Theory of the Learnable

advantageous. The question as to whether monotone DNF expressions can be learned from EXAMPLES alone is open. A positive answer would be especially signifi-

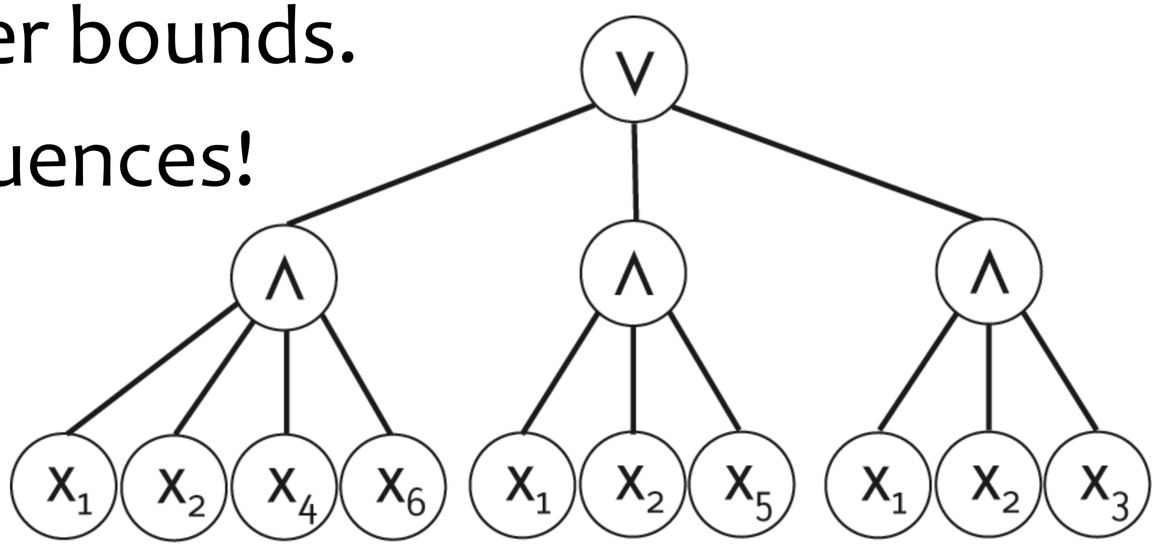
No Excuses!

Monotone juntas are easy.

MDNF can't compute parity.

No SQ lower bounds.

No consequences!



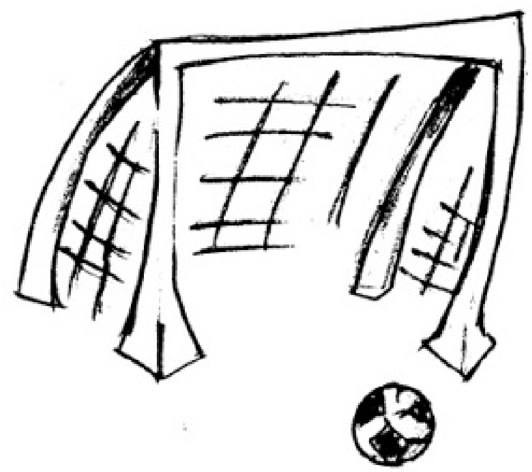
Known Results

- Poly(n)-size read-k MDNF. [HM91]
- Size-$2^{\sqrt{\log(n)}}$ MDNF [S01]
- Random poly(n)-size MDNF [S08,JLSW08]
  - Pick t terms uniformly from all terms of size log(t)
  - Relies on terms not overlapping too much

Pretty pitiful.

Setting a Goal

- Read-o(1)
- Size $\Omega(n)$
- Overlapping terms



Talagrand DNF

Pick n terms from set of all terms of length $\log(n)$ defined over first $\log^2(n)$ variables. [T96]

- Size n, read-o(1).
- Know all relevant variables.
- Lots of overlap.

Talagrand DNF

Pick n terms from set of all terms of length $\log(n)$ defined over first $\log^2(n)$ variables. [T96]

- f is sensitive to low noise

  $\Pr[f(x) \neq f(y)] = \Omega(1)$

  $y = x$ with each bit flipped with prob $1/\log(n)$
- f has high "surface area" $\Omega(\sqrt{\log(n)})$

Prizes

- PAC-learn Talagrand DNFs w.h.p. over the choice of DNF.
- PAC-learn Talagrand DNFs in the worst case.
- Prove that Talagrand DNFs require $n^{\omega(1)}$ SQ-queries [FLS10].